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ORTHOGONALLY STIFFENED PLATES

by

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Mechanics of Solids

Department of Mechanical Engineering

The Johns Hopkins University

Baltimore, Maryland

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NOMENCLATURE

ϵ_x = extensional strain in x direction

ϵ_y = extensional strain in y direction

γ_{xy} = shear strain in x, y plane

σ_x = extensional stress in x direction

σ_y = extensional stress in y direction

τ_{xy} = shear stress in x, y plane

S_{ij} = elastic constants ($S_{ij} = S_{ji}$)

$E_x = \frac{1}{S_{11}}$ = Young's modulus

$E_y = \frac{1}{S_{22}}$ = Young's modulus

$G_{xy} = \frac{1}{S_{66}}$ = rigidity or shear modulus

$\nu_{xy} = -E_x \cdot S_{12}$ = Poisson's ratio

$\nu_{yx} = -E_y \cdot S_{12}$ = Poisson's ratio

M_b = bending couple per unit length distributed uniformly on two opposite edges

M_t = twisting couple per unit length of edge of plate

W = deflection of plate

h = thickness of plate (equivalent orthotropic plate)

a = width of plate (bending)

b = length of plate (bending)

c = length of side of plate (twisting)

x, y = coordinates of point on surface of plate

A, B, C = constants of integration

A_x, A_y, A_{xy} = constants in differential equation of plate and are expressions in terms of the elastic constants S_{ij}

$F(x, y, t)$ = Force applied normally to face of plate

ρ = mass density

t = time

P_{mn} = circular frequency of m, n mode of vibration of rectangular plate

m, n = mode numbers corresponding to x -direction and y -direction respectively

P = load applied through lever to test plates in bending and in twisting

ORTHOGONALLY STIFFENED PLATESINTRODUCTION

Flexure theory for flat plates of homogeneous orthotropic material has been in existence for many years, (1)¹. Its extension to cover the case of plates with attached stiffeners was more recently introduced by several investigators (2,3,4,5). For this latter case the thickness of the plate is constant and the stiffeners in each of two orthogonal directions are identical, parallel, and fairly closely spaced. A repeating unit is regarded as an infinitesimal element of an equivalent orthotropic plate of corresponding stiffness factors or compliances. The entire composite structure will bend and twist approximately the same as a homogeneous orthotropic plate of equivalent stiffness. This type of analysis is particularly applicable for composite structures such as stiffened bottoms in ships and fuselages of airplanes (6, 7, 8, 9).

Examination of the technical literature shows that investigators invariably attempt to estimate the appropriate unit stiffness factors or compliances for stiffened plates on a theoretical basis, using beam theory as a guide. Cross-contraction effects associated with a Poisson's type ratio are usually neglected.

The present paper presents an experimental method for the determination of the effective stiffness moduli of the actual stiffened structure. Rectangular portions or "patches" of the plate containing a sufficient number of the stiffeners to be representative are subjected to pure bending and twisting couples distributed along their edges.

The effectiveness of the method is investigated by comparison of the fundamental frequency of the flexural vibrations of a simply supported stiffened plate calculated with the statically determined stiffness factors or compliances and then determined experimentally in vibration tests.

¹Numbers in parentheses refer to bibliography at end of paper

THEORY OF EQUIVALENT ORTHOTROPIC PLATE

A typical stiffened plate is shown in Fig. 1. For simplicity the stiffeners are shown only in one direction; however, there is no additional difficulty in the theory itself if there is a second set of stiffeners orthogonal to the first. In fact, for the plate shown in Fig. 1, the bending stiffness in the direction parallel to the stiffeners is obviously different from that in the direction transverse to the stiffeners. This plate has a repeating unit of equally spaced and identical stiffeners. The spacing is small compared with the length of a stiffener.

For the purpose of determining flexural displacements, the stiffened plate may be considered as if it were approximately a plate of homogeneous orthotropic material of some definite thickness. It can be shown that the elastic constants of this equivalent orthotropic plate can be chosen in such a manner that it will have approximately the bending and twisting stiffnesses or compliances of the given orthogonally stiffened plate.

Now suppose that conveniently sized but sufficiently representative rectangular and square test plates are fabricated just like sections from the stiffened prototype. These test plates may themselves be considered as small orthotropic plates. The rectangular models will serve for bending testings and the square ones for twisting tests. The elastic constants determined from test on the samples will be assumed to be those corresponding to the stiffened prototype.

It will be assumed further that the plates are effectively homogeneous and orthotropic and therefore have three axes of symmetry at each point. The test plates have their edges parallel to the stiffeners and hence parallel to the principal directions of stiffness.

In the usual notation (10) the stress-strain relations of the equivalent orthotropic material may be written for the strains in terms of the stresses and elastic constants as follows:

$$\left. \begin{aligned} \epsilon_x &= S_{11} \sigma_x + S_{12} \sigma_y = \frac{\sigma_x}{E_x} - \frac{\nu_{xy}}{E_x} \sigma_y \\ \epsilon_y &= S_{21} \sigma_x + S_{22} \sigma_y = -\frac{\nu_{yx}}{E_y} \sigma_x + \frac{\sigma_y}{E_y} \\ \gamma_{xy} &= S_{66} \tau_{xy} = \frac{\tau_{xy}}{G_{xy}} \end{aligned} \right\} \quad [1]$$

There are then four constants of orthotropic elasticity S_{1j} ($S_{12} = S_{21}$) or E_x , E_y , G_{xy} , and

$$\frac{\nu_{xy}}{E_x} = \frac{\nu_{yx}}{E_y}$$

Now the equation for equilibrium of an orthotropic rectangular plated loaded by couples on its boundary is (1,12):

$$A_x \frac{\partial^4 w}{\partial x^4} + A_{xy} \frac{\partial^4 w}{\partial x^2 \partial y^2} + A_y \frac{\partial^4 w}{\partial y^4} = 0 \quad [2]$$

where A_x , A_y , A_{xy} are constants in terms of the elastic constants S_{1j} of the stress-strain relations.

The bending deflection for a rectangular plate with couples M_b distributed on two opposite edges which are themselves perpendicular to one direction of principal stiffness is (12):

$$W = \frac{6M_b}{h^3} (S_{11}x^2 + S_{12}y^2) + Ax + By + C \dots [3]$$

The bending deflection for a second rectangular plate with couples M_b distributed on two opposite edges which are themselves perpendicular to the other direction of principal stiffness is:

$$W = \frac{6M_b}{h^3} (S_{22}x^2 + S_{21}y^2) + Ax + By + C \dots [4]$$

The twisting deflection for a square plate with stiffeners parallel to the two orthogonal edges respectively and twisting couples M_t distributed over all of the edges is:

$$W = \frac{6M_t}{h^3} \cdot S_{66} \cdot xy + Ax + By + C \dots [5]$$

It can readily be seen that since equations [3], [4], and [5] are quadratic in the variables x and y , they satisfy the equation of equilibrium [2] each of whose terms are of the fourth order. Also, it can easily be shown that these equations satisfy the boundary conditions of distributed couples. The constants of integration A , B , and C are determined from knowledge of the locations of the three support points of the test plate for the case of bending and from knowledge of the locations of two support points and a condition of symmetry for

the case of twisting.

It is obvious that in bending tests, one may measure W , M_b , x , and y in [3] for one principal direction or in [4] for the other principal direction and thereby obtain two simultaneous equations for the determination of the elastic constants S_{11} , S_{12} , and S_{22} in terms of the cube of the thickness. In twisting tests on square plate one may measure W , M_t , x , and y in [5] and thereby determine the shear constant S_{66} in terms of the cube of the thickness.

If the tests are performed on uniformly thick but unstiffened plates of orthotropic material the thickness could be explicitly determined by a single measurement (13). However, for stiffened plates, this quantity is not explicitly defined. The sole condition is that depth of plate plus stiffener shall be small compared to say the length of a side of the plate. This lack of necessity to explicitly specify a plate thickness or locate a neutral surface of bending for the plate are strong points in favor of the method proposed in this paper.

Once the elastic moduli for an equivalent orthotropic plate are determined by test, they may be used in calculating bending deflections for plates of identical stiffened construction but any given boundary conditions. Also, The determination of these elastic moduli by statical bending and twisting tests permits one to calculate the dynamic response of orthogonally stiffened plates by replacing the differential equation of statical equilibrium [2] by the equation of motion which will now be developed.

DIFFERENTIAL EQUATION OF MOTION FOR FLEXURE OF EQUIVALENT ORTHOTROPIC PLATE

The differential equation for the flexural vibrations of a thin plate of elastic orthotropic material of constant thickness was established many years ago (1). Its use in making an experimental study of the fundamental frequency of vibration of unstiffened wood and plywood plates of constant thickness was investigated less than ten years ago (14). This theory will be briefly restated and appropriately inter-

puted for use in connection with the present problem of the vibration of plates with stiffeners attached.

The differential equation [2] can be readily modified to allow for external forces F applied normally to the plate (10) .

Then it becomes:

$$A_x \frac{\partial^4 W}{\partial x^4} + A_{xy} \frac{\partial^4 W}{\partial x^2 \partial y^2} + A_y \frac{\partial^4 W}{\partial y^4} = F(x, y) \quad [6]$$

As usual the differential equation for the flexural vibration of the plate, if F becomes a function of time, can be obtained by adding the effective inertia force $(-\rho h \frac{\partial^2 W}{\partial t^2})$ to the applied external force F so that the equation becomes:

$$A_x \frac{\partial^4 W}{\partial x^4} + A_{xy} \frac{\partial^4 W}{\partial x^2 \partial y^2} + A_y \frac{\partial^4 W}{\partial y^4} + \rho h \frac{\partial^2 W}{\partial t^2} = F(x, y, t) \quad [7]$$

As remarked in connection with [2], the constants A_x , A_y , A_{xy} can be put in terms of the elastic constants S_{ij} . For this purpose, we solve the stress-strain equations for stress in terms of strain and obtain (10):

$$\left. \begin{aligned} \sigma_x &= E'_x \epsilon_x + E'' \epsilon_y \\ \sigma_y &= E'_y \epsilon_y + E'' \epsilon_x \\ \tau_{xy} &= G_{xy} \delta_{xy} \end{aligned} \right\} \text{where} \quad \left. \begin{aligned} E'_x &= \frac{S_{22}}{S_{11} S_{22} - S_{12}^2} \\ E'_y &= \frac{S_{11}}{S_{11} S_{22} - S_{12}^2} \\ E'' &= \frac{S_{12}}{S_{12}^2 - S_{11} S_{22}} \\ G &= \frac{1}{S_{66}} = G_{xy} \end{aligned} \right\} \text{--- [8]}$$

For a simply supported rectangular plate the solution of [7] when $F \equiv 0$ is as can easily be seen:

$$W = \sum \sum A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \dots [9]$$

Substitution of [9] into [7] with $F \equiv 0$ gives the equation for the circular frequency P_{mn} as follows:

$$P_{mn} = \frac{\pi^2}{\sqrt{\rho h}} \sqrt{D_x \frac{m^4}{a^4} + 2H \frac{m^2 n^2}{a^2 b^2} + D_y \frac{n^4}{b^4}} \dots [10]$$

$$D_x = \frac{E'_x h^3}{12}$$

$$D_y = \frac{E'_y h^3}{12}$$

$$D_1 = \frac{E'' h^3}{12}$$

$$D_{xy} = \frac{G_{xy} h^3}{12}, \text{ and } H = D_1 + 2D_{xy}$$

For the fundamental mode m and n each equal unity and for a square plate, a is equal to b .

It is important to note that for the case of the stiffened plate, the S_{ij} in equations [3], [4], and [5] are proportional to the cube of the thickness h . This fact together with relations [8] show that the quantities E'_x , E'_y , E'' , and G_{xy} vary inversely as the cube of the thickness h . Hence the quantities D_x , D_y , D_1 and D_{xy} , and H in [10] are independent of the thickness h .

Furthermore, the product ρh which occurs in [10] and in the differential equation of motion [7] can be evaluated by weighing the plate and dividing by the area of the plate. This is so because in the equation of flexural vibrations, the term ρh is simply the mass per unit area of the plate.

With these facts, the fundamental frequency of the orthogonally stiffened plate can be calculated.

EXPERIMENTAL DETERMINATION OF ORTHOTROPIC ELASTIC

CONSTANTS OF ORTHOGONALLY STIFFENED PLATE

The experimental method used to determine the elastic constants is an adaptation of one used by Bergsträsser (11) for isotropic plates and by Hearmon (12) for unstiffened orthotropic plates of constant thickness.

In the bending tests, the equivalent of a uniformly distributed M_y must be applied to two opposite sides of a plate. In Fig. 5 the schematic arrangement of the plate resting on three supports is shown. The actual test arrangement with loading device to produce the moment is shown in Fig. 7. Between the supports the assumption, of course, is that the bending moment is uniformly distributed across the plate.

For the twisting tests, the equivalent of a uniformly distributed twisting couple L_t around the entire boundary must be provided. This can be obtained (11) as shown schematically in Fig. 5 by supporting the plate on two opposite vertices and loading on the two remaining vertices. The actual test arrangement with loading device to produce the twisting couple is shown in Fig. 6.

From the theoretical formulation it is clear what measurements must be made to determine the elastic constants. For the case of bending it is sufficient to measure the deflections at the origin of coordinates and at a short distance off center say along the y-axis shown in Fig. 5. For the determination of the shearing modulus G_{xy} , it is clear that only a single deflection is required and this is conveniently measured at the origin of coordinates shown on Fig. 5 .

For the bending tests one rectangular stiffened plate is fabricated with its long edge parallel to one set of the stiffeners and another rectangular stiffened plate with its long edge parallel to the other set of stiffeners. The two principal directions of stiffness are thereby orthogonal. For the twisting test a square stiffened plate is fabricated with one set of its stiffeners parallel with one edge of the plate.

For the purpose of experimentally studying the methods proposed in the present paper the types of stiffened test plates were selected with proper regard for economy of fabrication. However, it is considered that they provide useful information about the nature of results to be expected in the general case.

The plate material used was cold rolled steel about one-eighth of an inch thick. It was ground on both sides to a thickness of 0.108 inches. To satisfy the requirements previously mentioned, two plates were made for bending and one for twisting. Two types of orthogonal stiffeners were provided by milling grooves in one side of the plate for one type and by silver soldering one-eighth inch diameter brass rods to one side of the plate for the other type. The two patterns with dimensions are shown in Fig. 2. Photographs of the actual plates are shown in Fig. 3 and in Fig. 4. The bending plates were approximately $10\frac{1}{2}$ " long by $5\frac{1}{4}$ " wide while the twisting plates were approximately $5\frac{1}{4}$ " on each side.

For the purpose of loading the plates, a lever was used as shown in Fig. 7. This lever provides a known load on an auxiliary bar or plate suspended beneath and parallel to the test plate. The auxiliary plate in the bending tests provides forces on two opposite ends of the test plate, and within the region between supports on the test plate approximately uniform moment per unit length is distributed across the width of the plate. In the case of the twisting test an auxiliary bar was suspended from two opposite corners of the test plate and loaded at its center by a lever.

The load was gradually increased and the corresponding deflection measured at the chosen point on the plate. The results give a load-deflection curve for that particular point. For a linear relation, it is the slope of this line that gives a deflection per unit load, $\left[\frac{\delta}{W}\right]$, for use with equations [3], [4], and [5] in determining the equivalent elastic constants.

The deflections of the plates were referred to an absolute rigid frame shown in Fig. 6 and Fig. 7. A rigid pipe flange of 12 inches inside diameter was machined

to take a rotatable ground bar on which a sliding steel block is mounted. To this block, the deflection gage or micrometer is attached. The pipe flange is supported on top of three equi-height columns of sufficient stiffness. The rotatable bar can be rotated around the center of the pipe flange and the block holding the micrometer fixed can be slid axialwise along the smooth bar. In this manner, the deflection at any point can be measured.

So that the micrometer is not pushed hard against the test plate when measurements are being made, an electric contact indicator with glow tube is used. This is a pilot indicator with both lamp and magnetic relay which indicate when contact is just made. The setup is shown in Fig. 6 and Fig. 7.

Measurements were usually made at two points on the y-axis in addition to those at the center for the bending tests. The two side point values are averaged. For both bending and twisting, measurements were made for each side of the plate. First one side was up towards the measuring instrument and then the plate was turned over and the test repeated.

The force-deflection curves, from which are derived the moment deflection relations, are plotted in Figs. 8 to 17 inclusive. These curves represent twisting data for one plate each of the two types of stiffeners and bending data for two orthogonal directions each of the two types of stiffeners.

The bending deflections are for the center point of the plate and for points both above and below the x-axis on the y-axis. Both of these measurements above and below the x-axis are not required for determination of constants, but it is considered that their average gives a better result.

The results of computing the elastic constants are given in the following table.

Table I

ELASTIC CONSTANTS					
Type of Stiffener	$\frac{S_{11}}{h^3} \times 10^6$	$\frac{S_{12}}{h^3} \times 10^6$	$\frac{S_{21}}{h^3} \times 10^6$	$\frac{S_{22}}{h^3} \times 10^6$	$\frac{S_{44}}{h^3} \times 10^6$
Brass Rod Stiffener	9.67	0.06	- 4.95	13.9	49.5

Notice that $S_{12} \neq S_{21}$. The discrepancy is greater in the case of the brass rod stiffened plate. However, it is considered that the plate deflections are not sensitive to this constant and that an average value of S_{12} and S_{21} is quite acceptable. So for the brass rod stiffened plate the average of S_{12} and S_{21} is $-2.5h^3 \times 10^{-6}$ whence ν_{xy} is 0.26 and ν_{yx} is 0.18, reasonable values. And for the grooved plate the average of S_{12} and S_{21} is $-8.93h^3 \times 10^{-6}$. Whence ν_{xy} is 0.27 and ν_{yx} is 0.40.

From Table I it may be noted that there is considerable difference between S_{11} , the constant corresponding to one principal direction, and S_{22} , the constant corresponding to the other principal direction.

While it is obvious that the theory proposed in this paper can be checked by a study of the deflections at each point of the bent or twisted plates, or by loading plates statically with various boundary constraints, it was decided to make the test a dynamic one and compare the vibration frequencies of simply supported stiffened plates which are calculated from theory using the statically determined elastic constants and measured frequencies from the plates driven at resonance by an electromagnet.

EXPERIMENTAL DETERMINATION OF FUNDAMENTAL FREQUENCY OF VIBRATION OF STIFFENED PLATES

In order to determine the fundamental frequency of flexural vibration of the stiffened plates, a box-like rigid steel supporting frame was constructed as shown in Fig. 18. The boundaries of the plates were simply supported in these experiments. They were devised in a manner explained at length in a previous paper (15). Briefly, each plate for about three-eighths inch from its edge was ground flat and a 90° V-notch groove of depth equal to about 80% of the plate thickness was machined at the innermost boundary of the flat ground edge. A plate with grooved edge is shown fastened in the holding frame in Fig. 18. Outside the V-notched groove the flat edge of the plate is held rigidly on all four sides. It has been amply demonstrated (15) that this boundary condition is essentially that of a

hinged or simply supported plate.

Vibrations were excited in the mounted plate by an electromagnet solidly mounted just below the center of the plate. The magnet was driven by a Hewlett-Packard audio oscillator directly, with frequency being slowly increased until the first resonance condition could be detected. The resonance peak could approximately be detected acoustically by ear but in the experiments it was precisely determined with a crystal type phonograph pickup gently touching the plate. The output from the pickup was fed to a cathode ray oscilloscope where the resonance peak could be readily observed.

A comparison of the calculated and observed frequencies are shown in Table II. Data for an unstiffened plate which is practically isotropic are also included. This plate was ground to a thickness of 0.108 inches. As previously mentioned, it was a plate of this thickness which was grooved according to a definite pattern to produce one type of stiffened plate and had brass rods silver soldered to it to produce the other type of stiffened plate.

Table II

FUNDAMENTAL FREQUENCY OF PLATE IN C.P.S.		
Test Plate	Calculated	Observed
Isotropic (unstiffened)	1050	1050
Grooved pattern	980	960
Brass rod stiffeners	1120	1060

DISCUSSION AND CONCLUSIONS

As a result of this investigation, a method has been devised for the determination of the stiffness factors or compliances of an orthogonally stiffened plate. A previously used method (13) for determining the elastic constants of homogeneous materials by bending and twisting experiments on plates of constant thickness has been adapted for the purpose.

For both of two different types of orthogonally stiffened plates, grooved and rod stiffened, stiffness factors in the two principal orthogonal directions have been shown to differ by as much as 50 per cent.

An averaging procedure has been used to obtain approximate Poisson type cross-contraction ratios. The ratios determined in this manner seem reasonable.

The bending data for the brass rod stiffened plates are not as consistent as those for the accurately machined grooved plates as might be expected. The silver-soldering process used on the brass rods has some of the same uncontrollable features as electric welding in the fabrication of built-up structures. As a result, the brass rod stiffened plates were slightly warped. Averaging deflection measurements obtained from both sides of the plate, however, appear to minimize the effect of this deviation of the plate from a plane.

It is desirable to experiment with plates having larger stiffeners than those used in the present investigation but the dictates of economy necessarily limited the scope of the tests. Notwithstanding this fact, the chosen type of stiffeners seem to clearly illustrate the major point of the investigation; that is, the bending and twisting tests give reliable elastic constants for calculating flexural behavior of orthogonally stiffened plates under various loads.

It appears that good agreement exists between the calculated and the observed frequencies of vibration of the fundamental mode of each of the plates tested. The variations of the frequencies are not so impressive as are the variations for the elastic moduli among the various plates. The theoretical reason for this, however, is readily understood by observing the square root effect characteristic

of the frequency formula. As a result, the frequency of vibration is not as sensitive a physical variable as are the corresponding stiffness factors or compliances.

The vibration experiments appear to give a good confirmation of the proposed method of obtaining the equivalent elastic constants of stiffened plates.

The agreement between the theoretical and observed frequencies of the unstiffened plate serves as a verification of the assumption that the boundaries of the plates are simply supported.

The method of producing simply supported boundary conditions seems to be more suitable than those noted in the scanty literature on the subject. An example of another method for obtaining a simply supported plate is given in a paper by Hearmon on the fundamental frequency of wood plates of constant thickness (14).

It would be interesting to apply the method proposed in the present paper in order to determine the compliances of a stiffened plate in which the stiffeners are fairly deep.

The product of mass density and thickness of plate occurring in the pertinent equations of this paper can be directly determined as mass per unit area of plate by actually weighing a typical section of the plate and dividing by its area. Also, the proposed method for the determination of the effective elastic constants of the stiffened plate make it unnecessary to determine a thickness for the plate explicitly. These two facts combine to provide a straightforward method for the determination of the dynamical response of a stiffened plate. An example of this fact is presented in the paper in the form of a calculation of the fundamental mode of flexural vibration of a stiffened plate.

ACKNOWLEDGEMENTS

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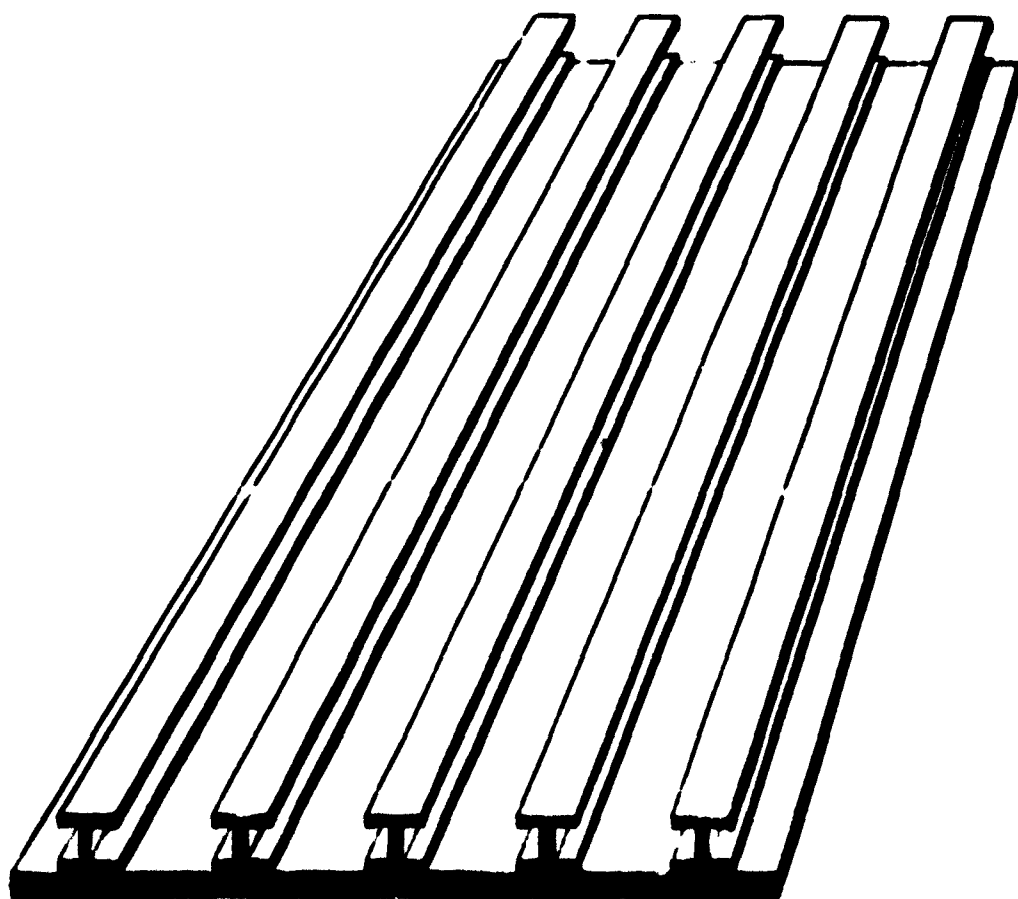
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TYPICAL STIFFENED PLATE

FIG. I

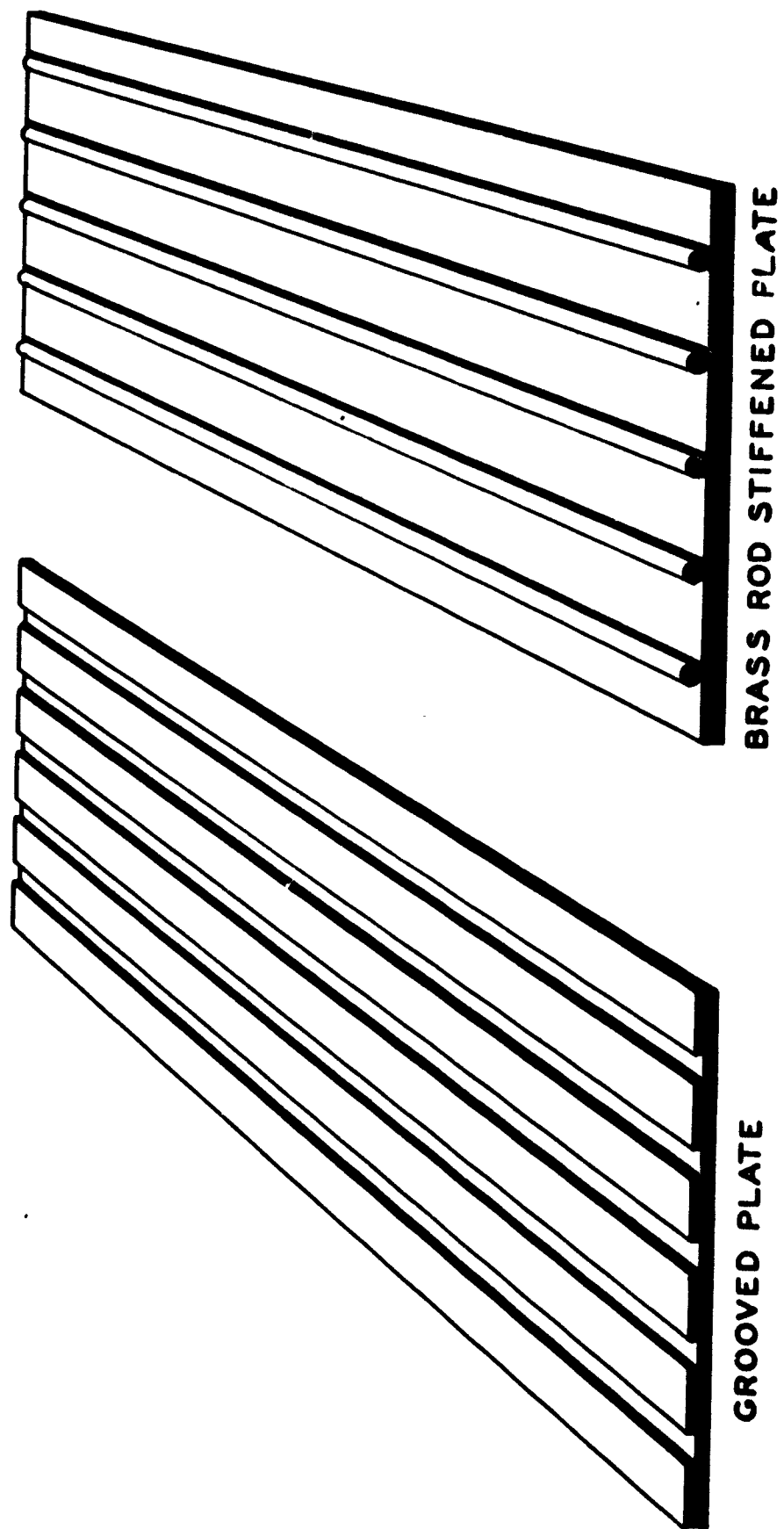


FIG. 2 STIFFENED EXPERIMENTAL PLATES

FIG. 3

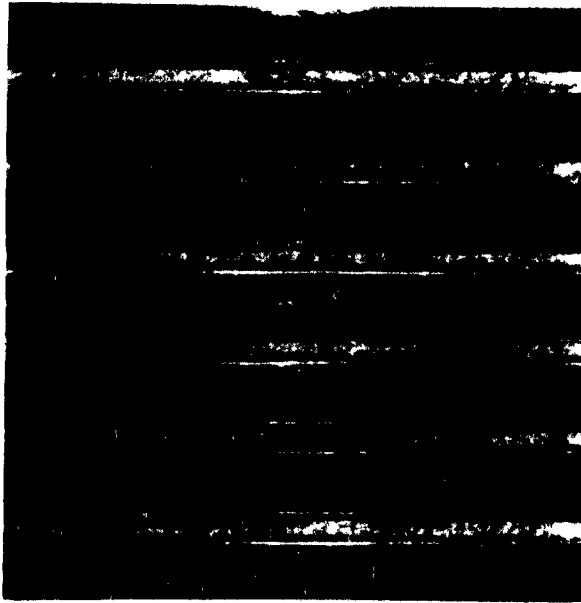


PLATE WITH
GROOVES

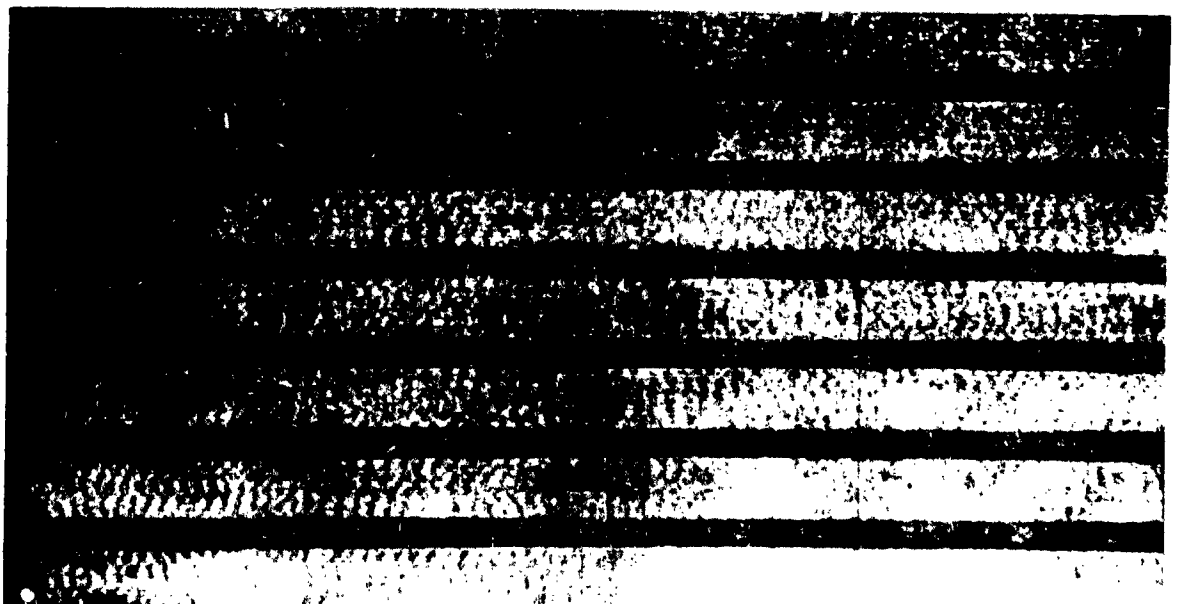


FIG. 4

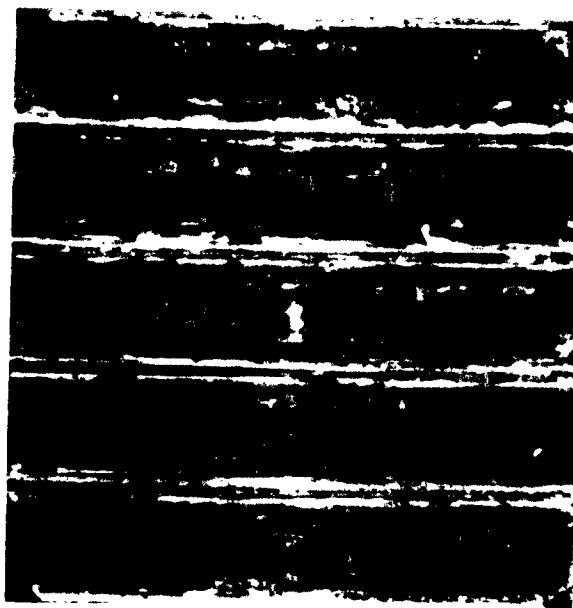
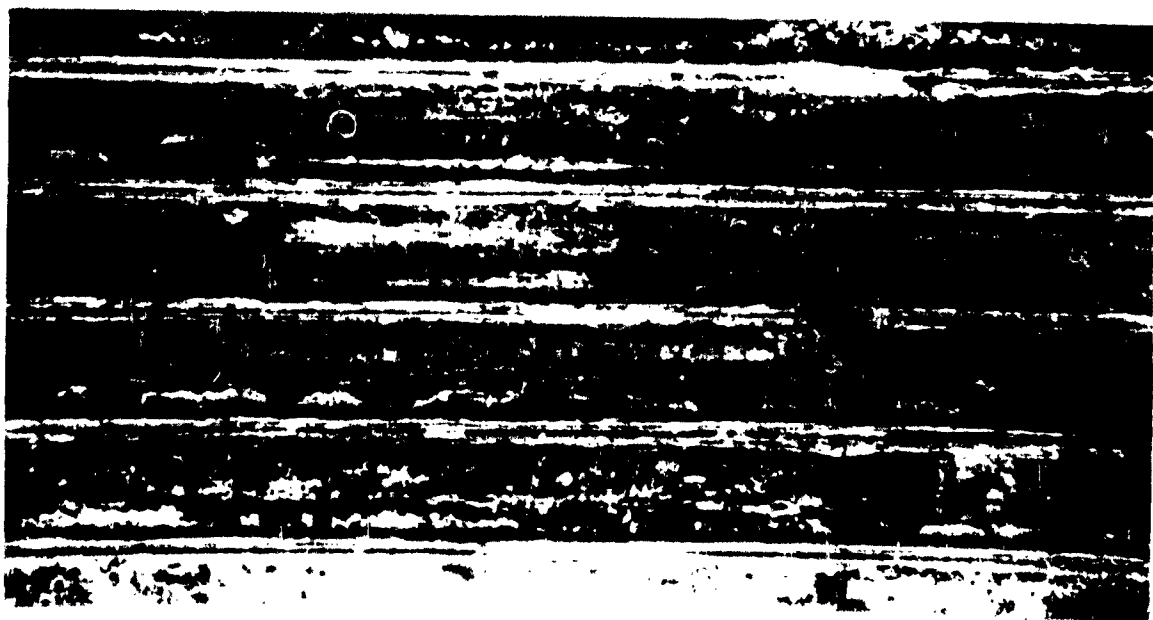
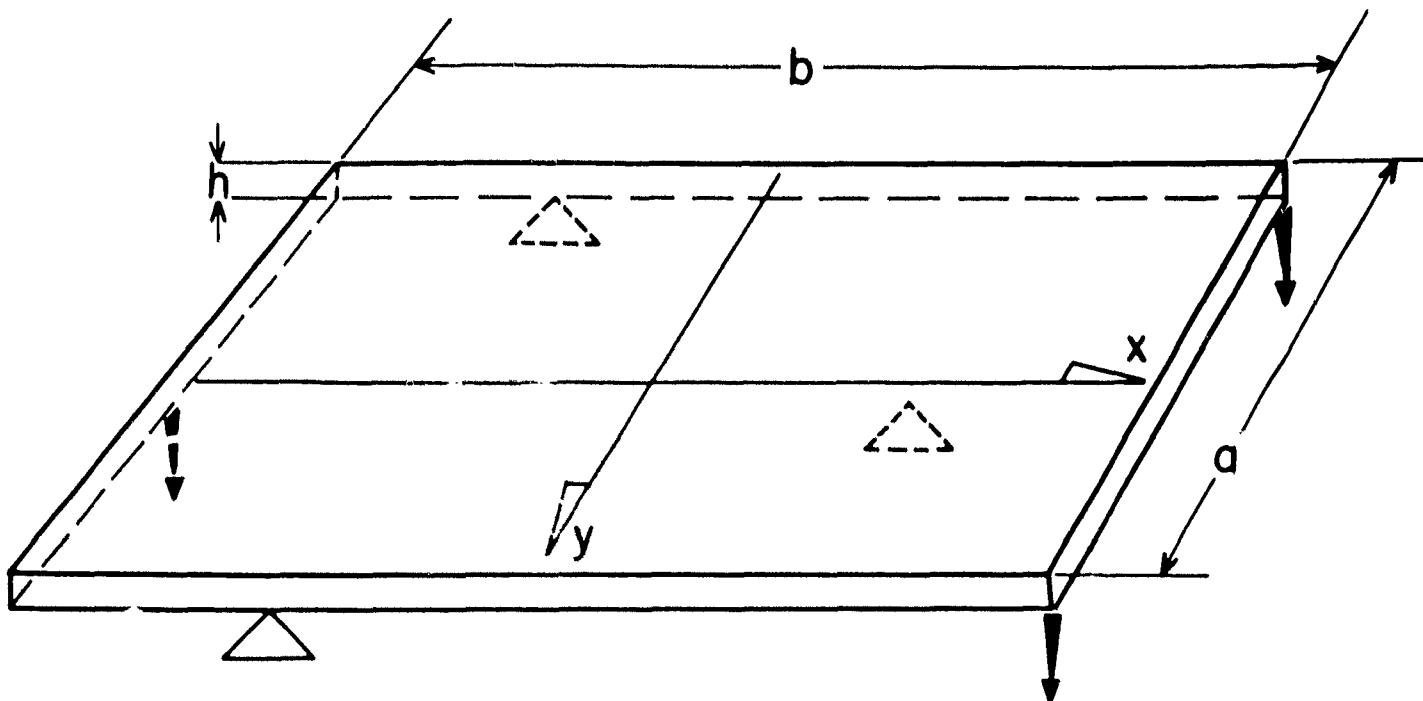
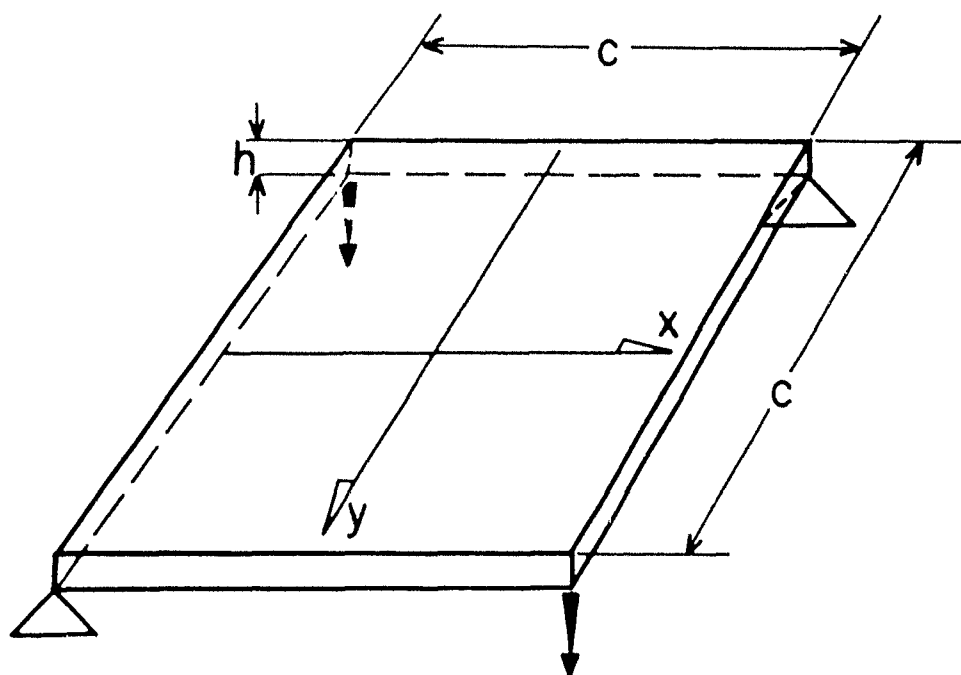


PLATE WITH
STIFFENERS



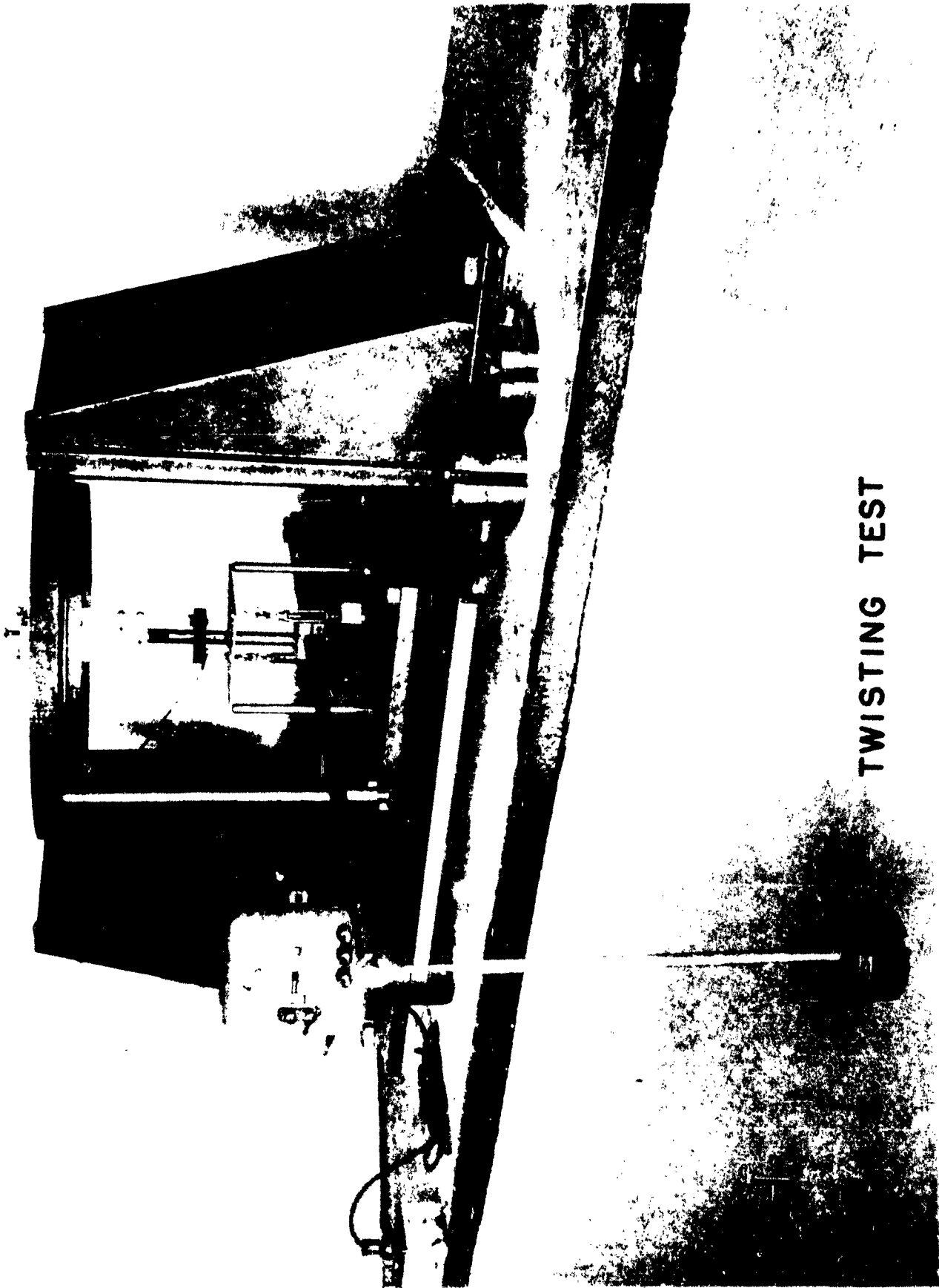


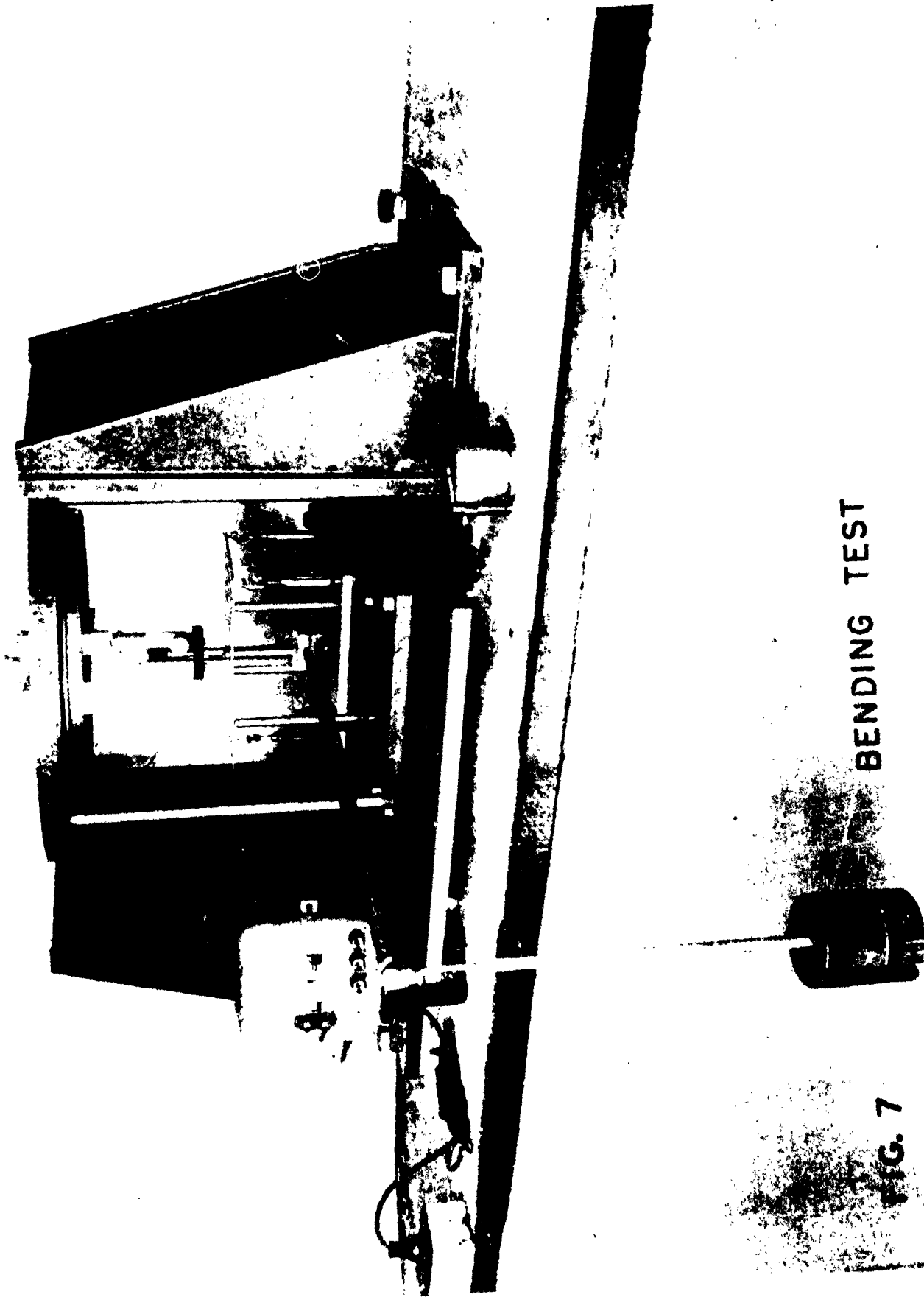
BENDING PLATE
FIG. 5 (A)



TWISTING PLATE
FIG. 5 (B)

TWISTING TEST





BENDING TEST

FIG. 7

"P" POUNDS

R-0

CENTER POINT READINGS

$\frac{P}{W}$ (LBS/INCH)

GROOVED SIDE UP

1300

GROOVED SIDE DOWN

1300



FIG. 8

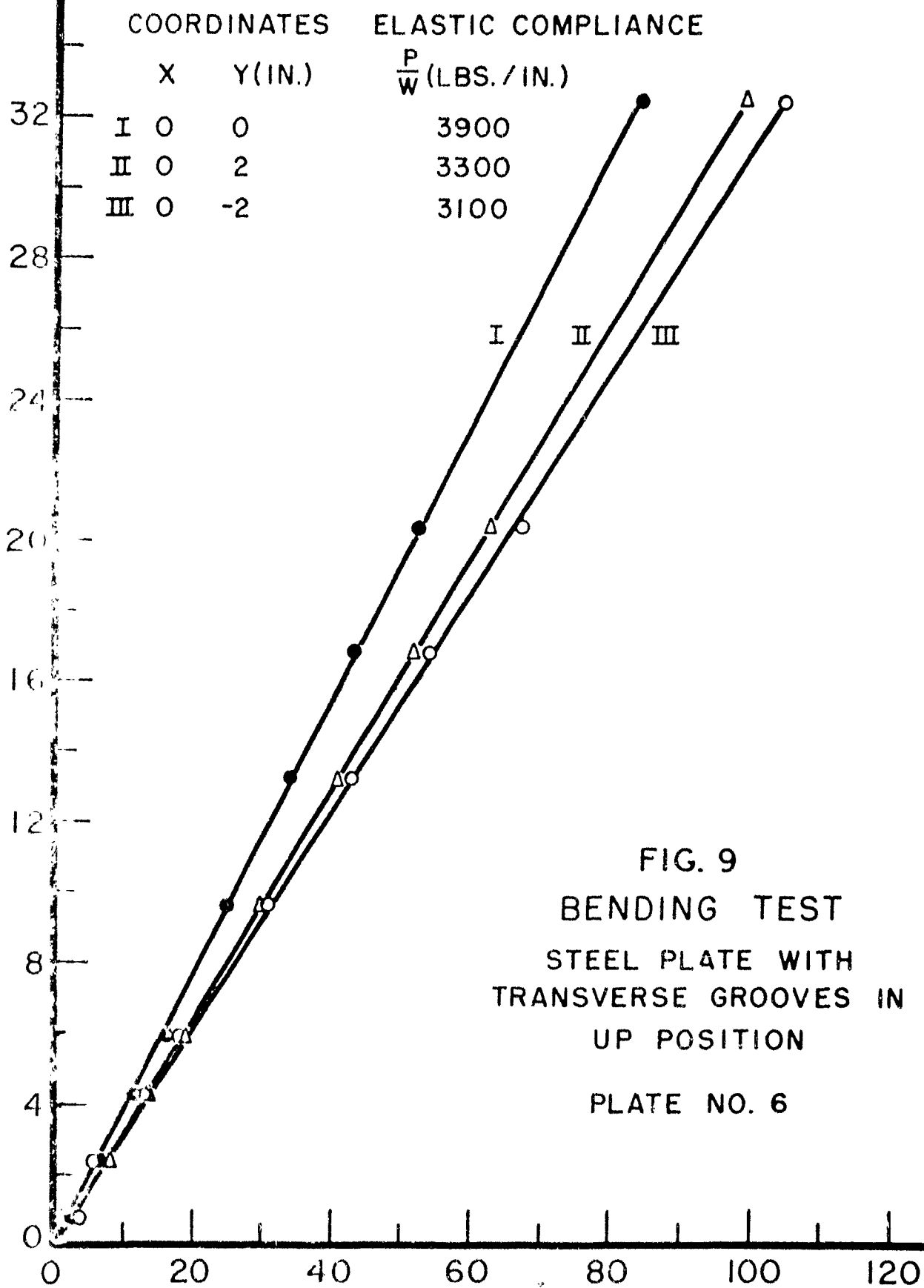
TWISTING TEST

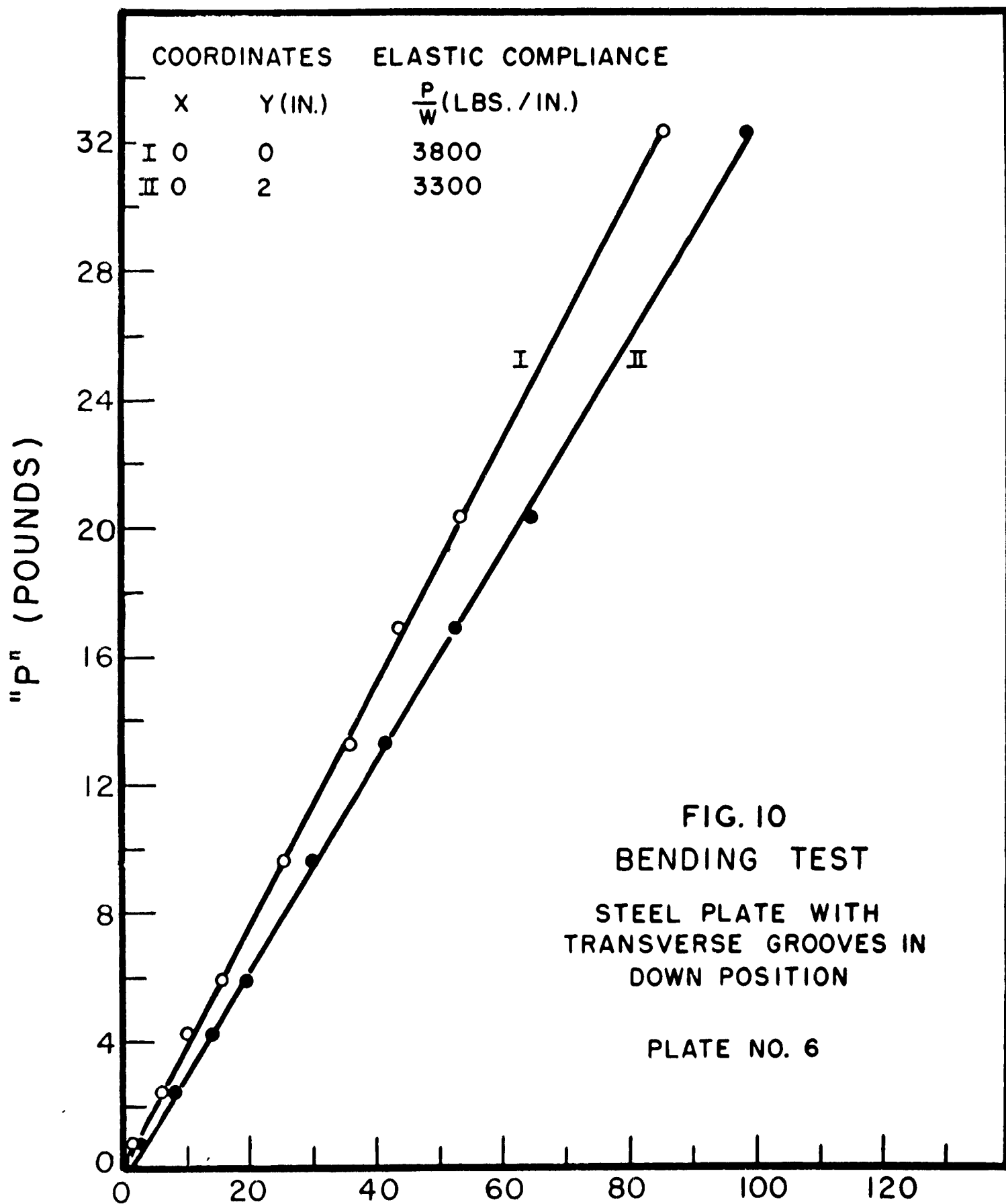
STEEL PLATE WITH
GROOVES

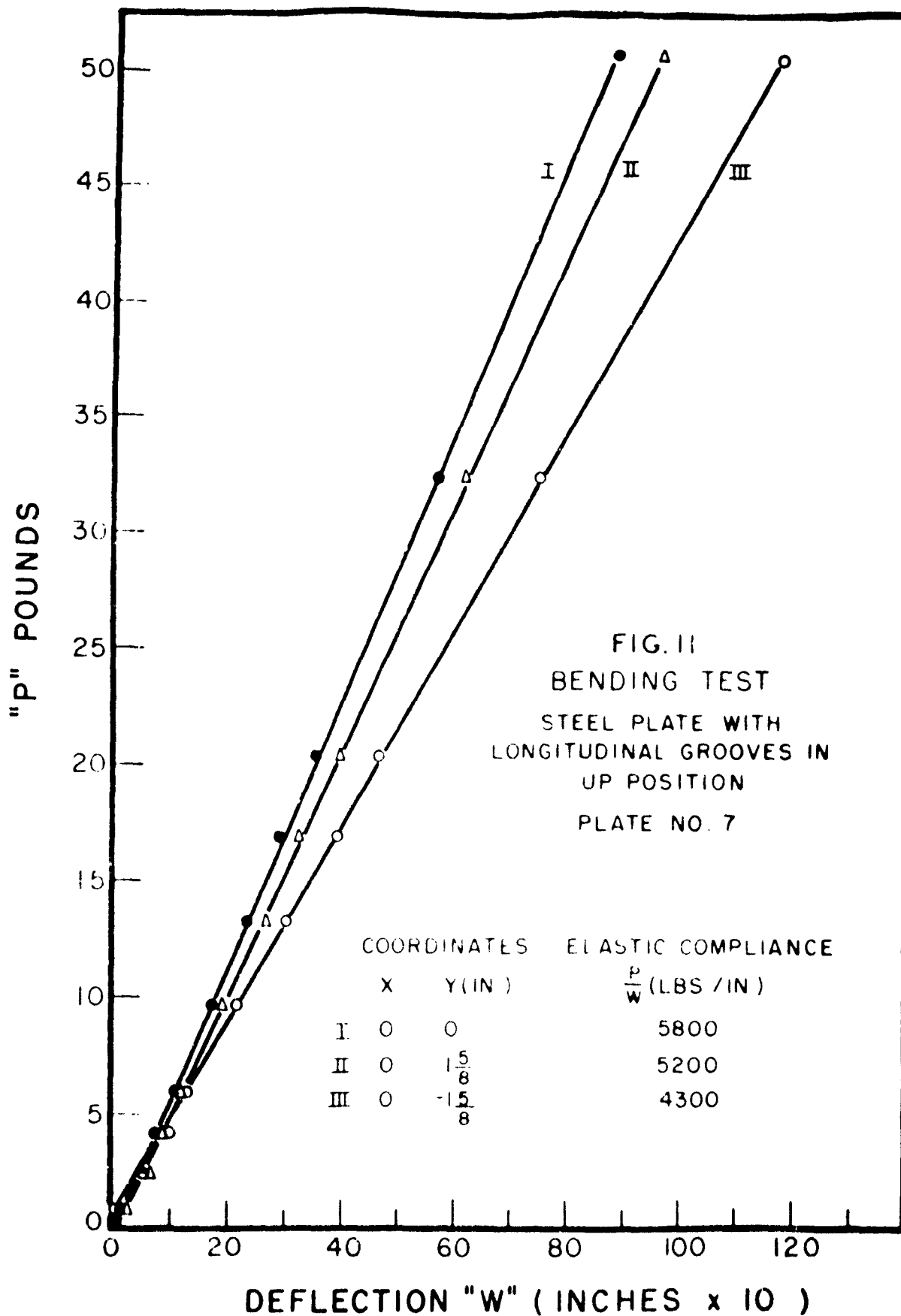
PLATE NO.8

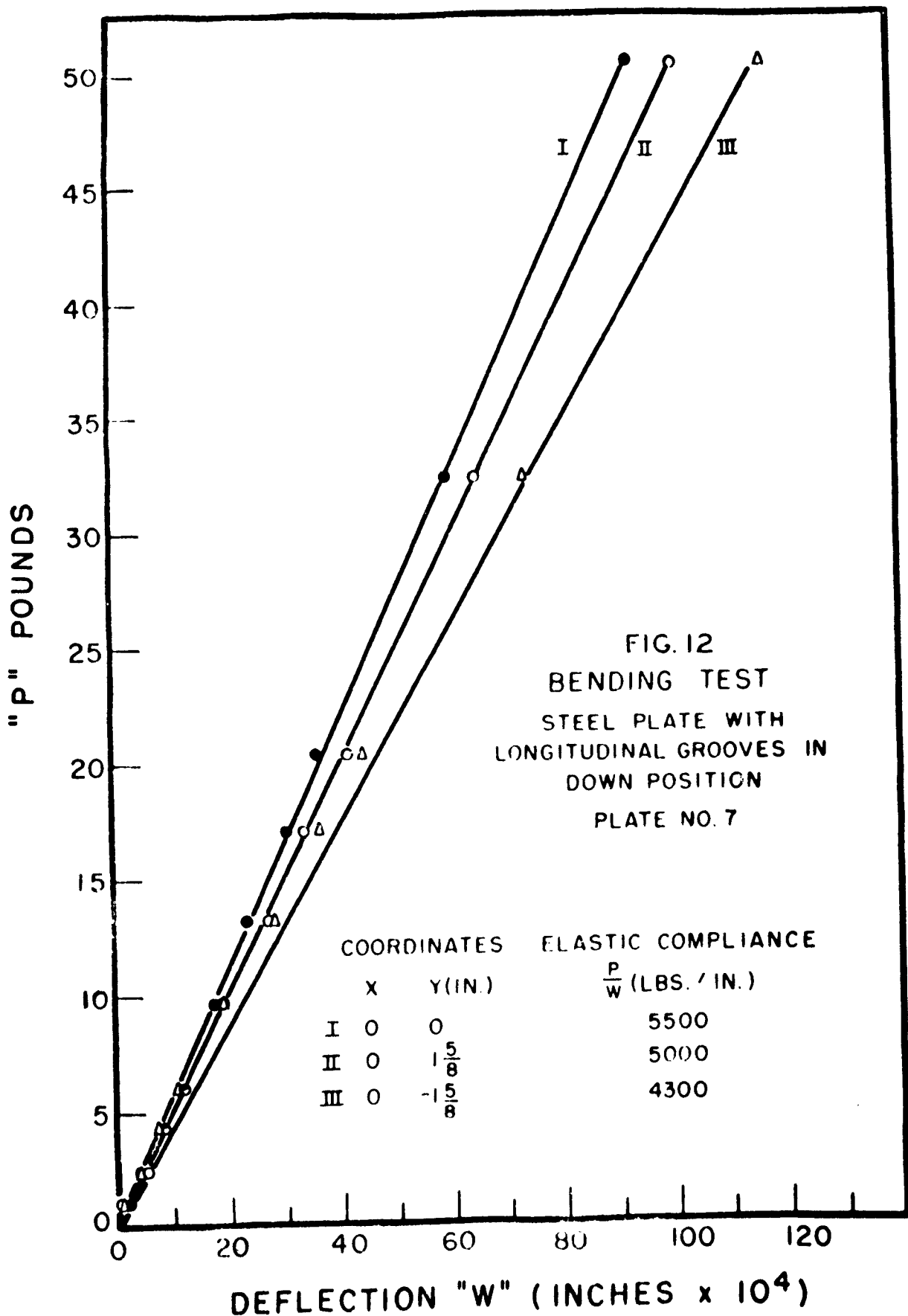
DEFLECTION "W" (INCHES)

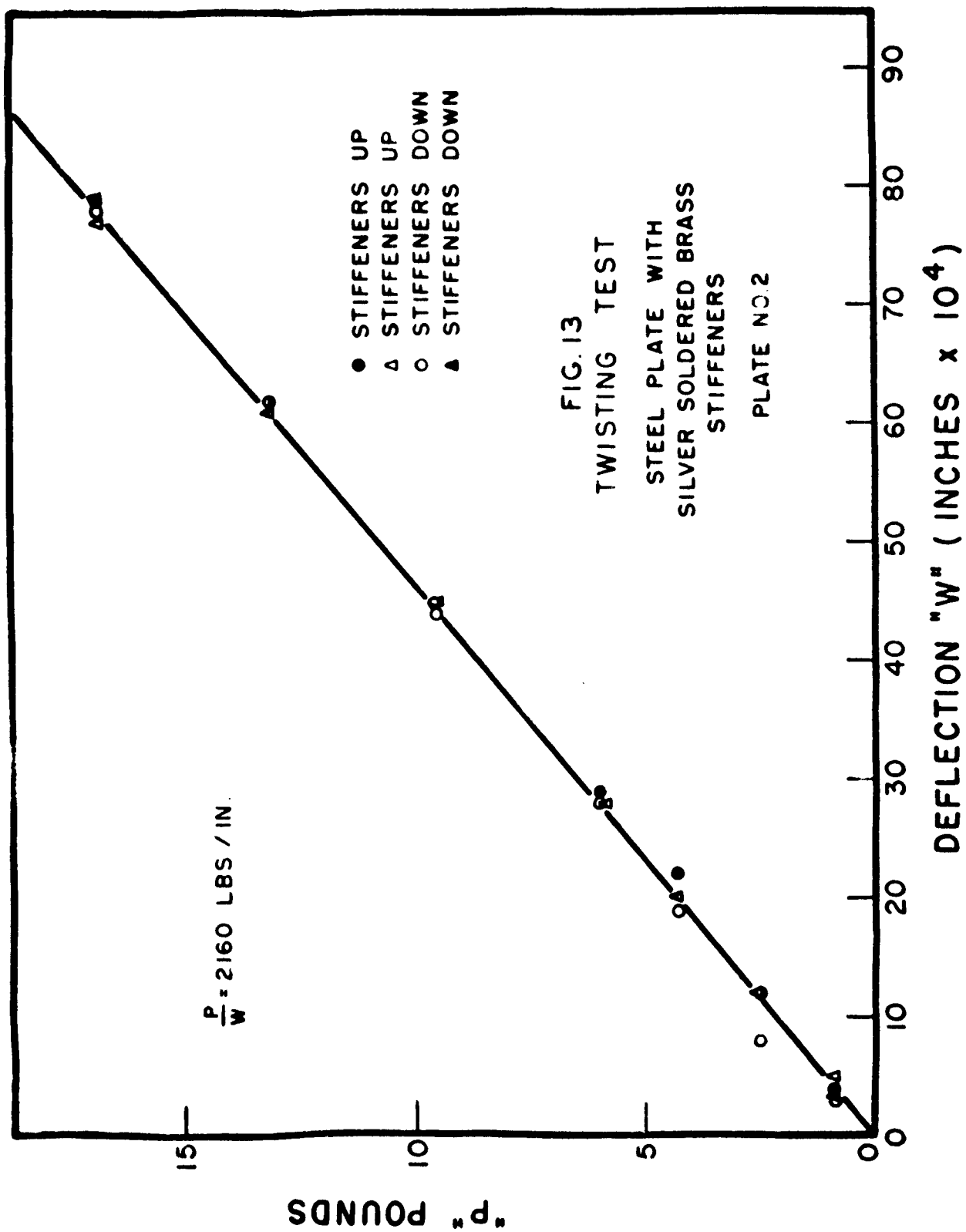
"P" POUNDS

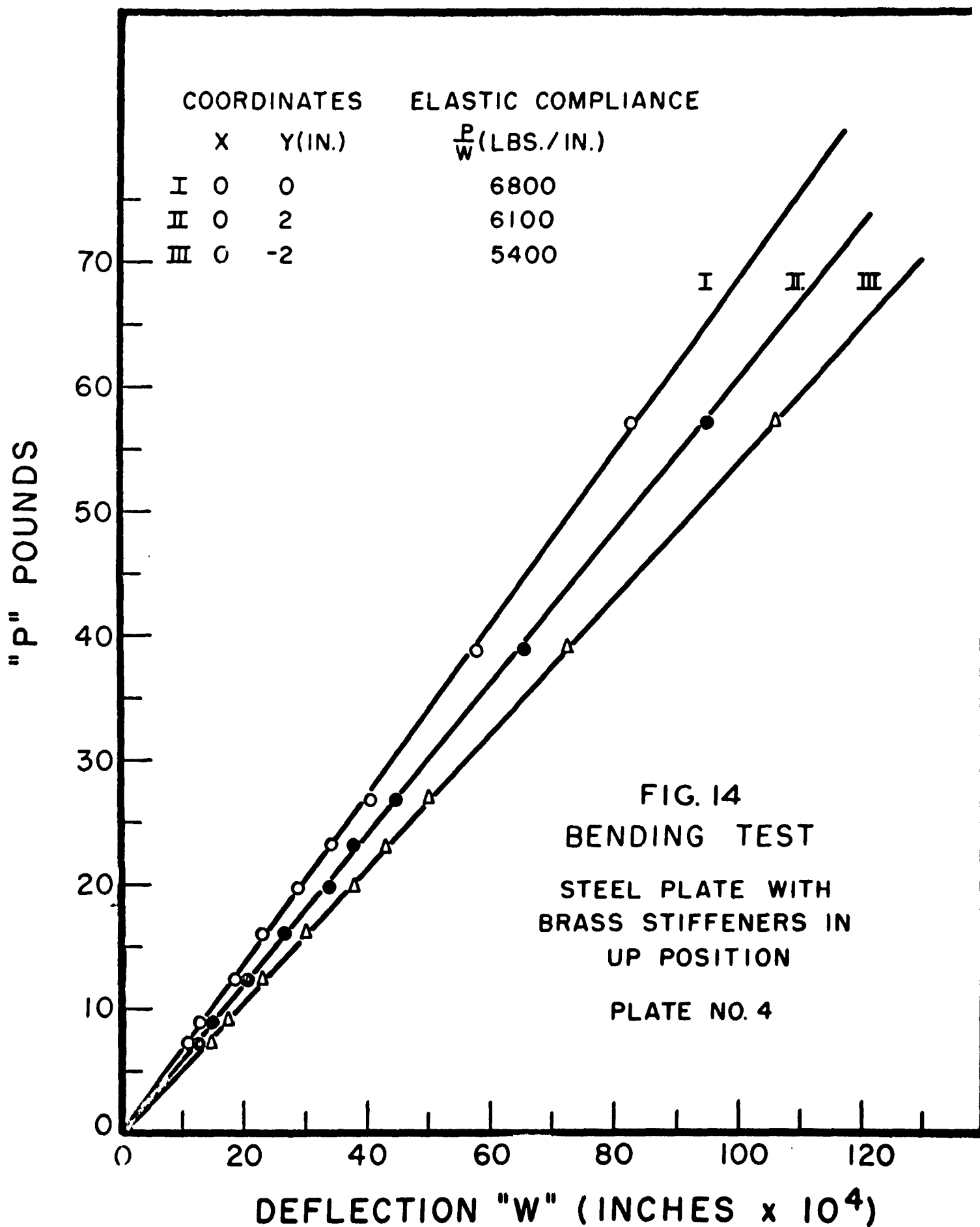


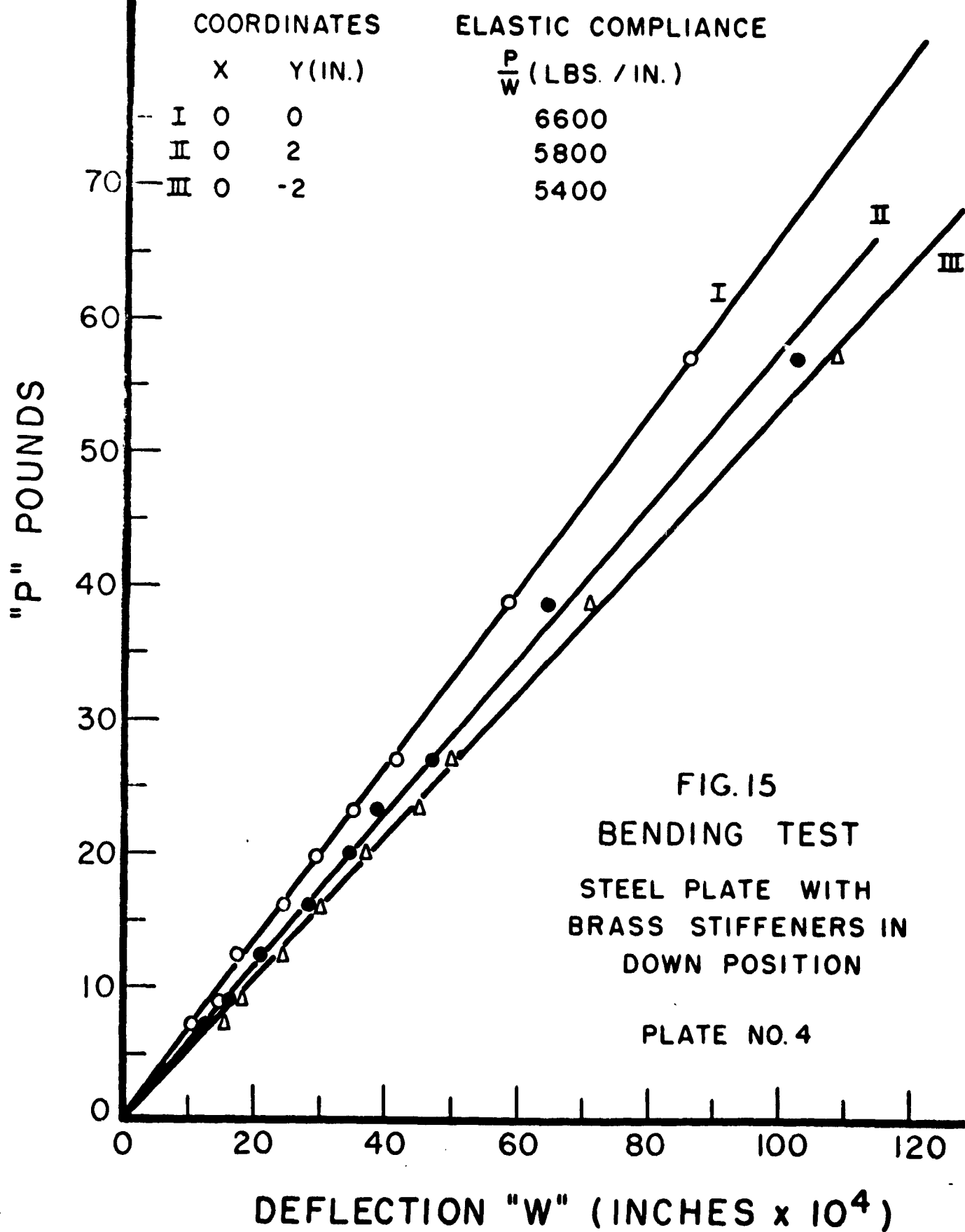






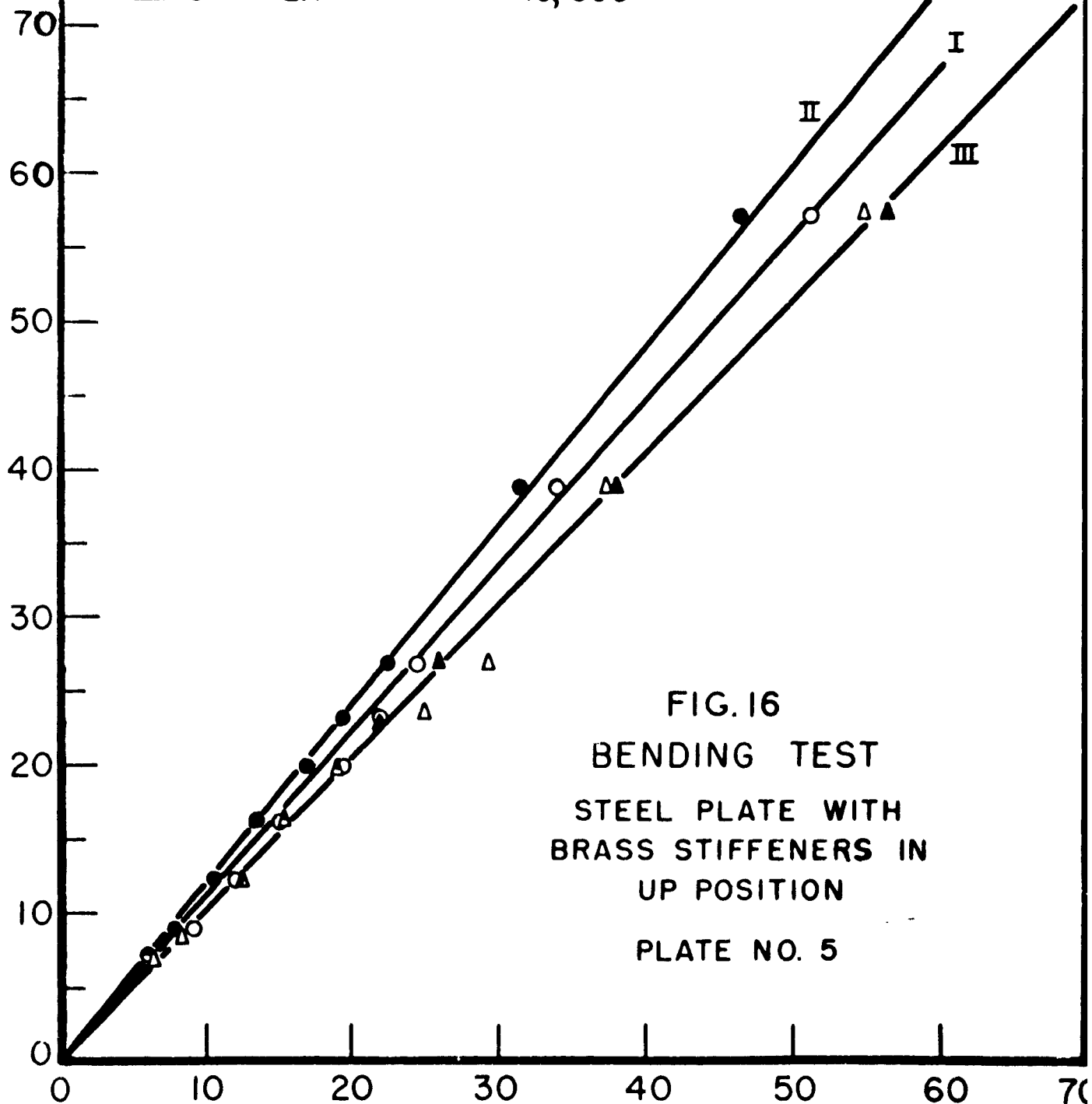




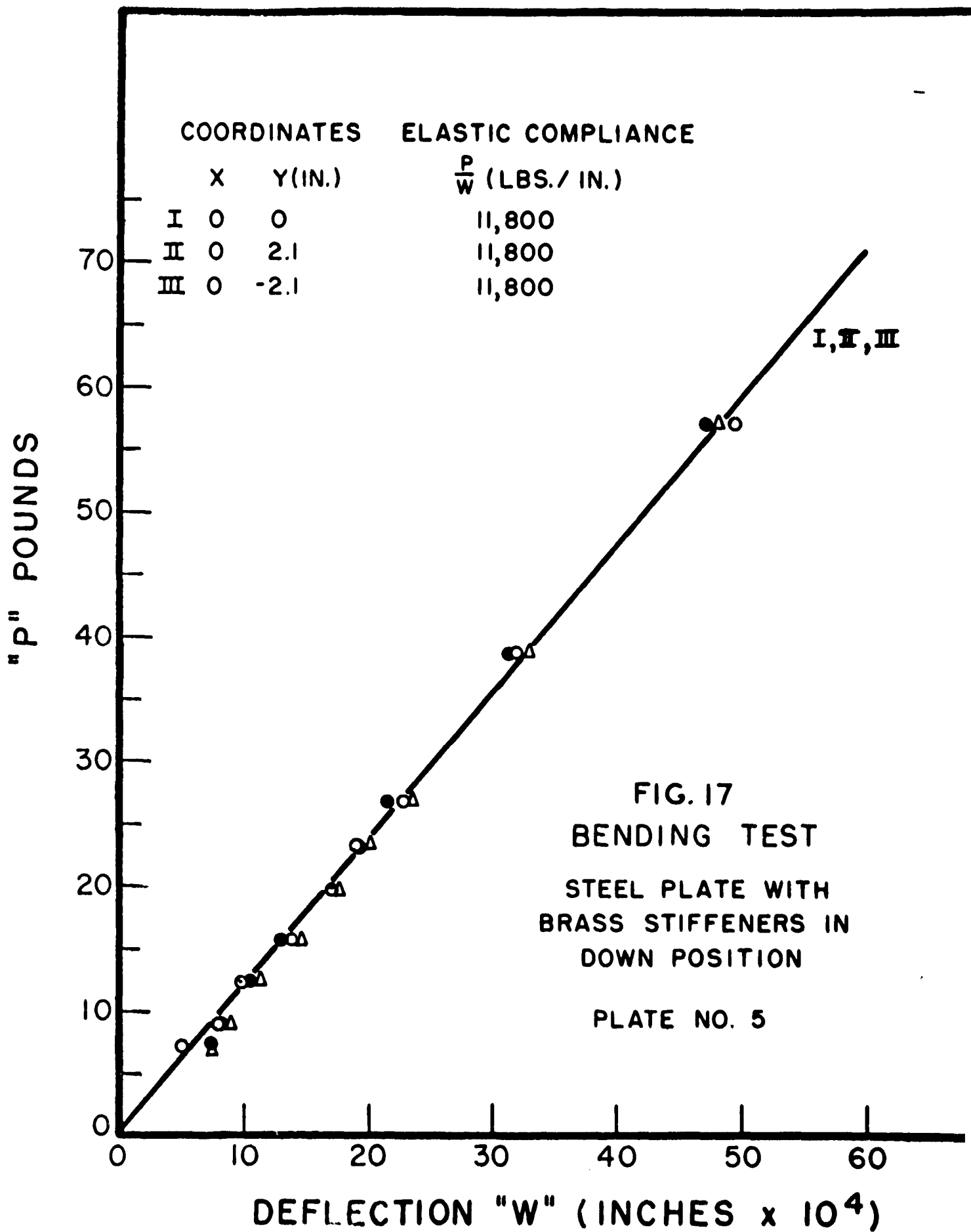


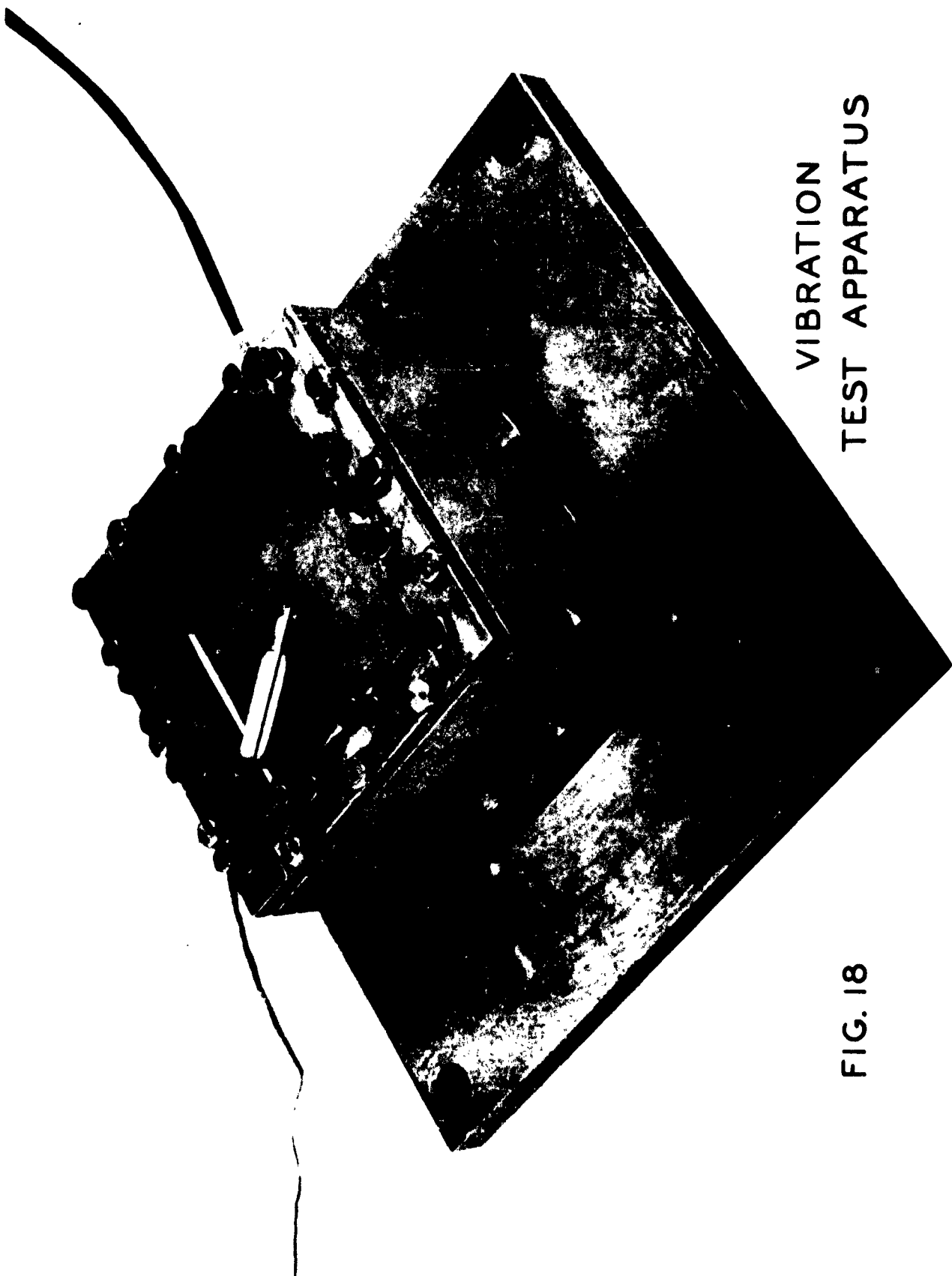
"P" POUNDS

COORDINATES			ELASTIC COMPLIANCE
	X	Y(IN.)	$\frac{P}{W}$ (LBS. / IN.)
I	0	0	11,100
II	0	2.1	12,100
III	0	-2.1	10,300



DEFLECTION "W" (INCHES x 10⁴)





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FIG. 18

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